Geometry. — On algebraic surfaces. Note by M. A. Clebsch, presented by M. Chasles.

«M. Riemann's theorems on algebraic functions of two variables have given a principle for classifying algebraic curves. I have proposed to call the *genus* of a curve the number $p = \dfrac{(n-1)(n-2)}{2} - d$ (the *deficiency* of M. Cayley), n being the order of the curve, d the number of its double points or cusps. The genus of two algebraic curves must be the same in order that one may make a single point of the other correspond to each point of the one, and conversely. All curves of the same genus can be transformed algebraically into a particular species of that genus, which has a sufficient number of arbitrary coefficients; for $p > 2$, these *normal curves* are of the order $p + 1$, and the number of their double points or cusps is equal to $\dfrac{p(p-3)}{2}$. (*See Clebsch und Gordan, Theorie der Abelschen Functionen.*)

»Now I have found that, for algebraic surfaces, there are entirely analogous theorems. I have managed to prove the following theorem:

«Let there be given two surfaces $f = 0$ and $\varphi = 0$ of the orders m and n respectively. Suppose that one may make a single point of $\varphi = 0$ correspond to each point of $f = 0$, and conversely, so that these surfaces are transformable one into the other in an algebraic and rational manner. Suppose, for greater simplicity, that these surfaces have only regular singularities, that is to say singularities which lie on each surface or on its reciprocal. Then let p be the number of arbitrary coefficients of a surface of the order $n - 4$ passing through the double or cuspidal curves which lie on $f = 0$, and p' the corresponding number in respect of $\varphi = 0$. One will always have $p' = p$.»

»This theorem allows us to classify these surfaces in respect of their *genus* p. Two surfaces must belong to the same genus in order to be transformed one into the other in a rational manner. When a given surface has no double or cuspidal curve at all, its genus will be equal to $\dfrac{(n-1)(n-2)(n-3)}{1\,.\,2\,.\,3}$, all the coefficients of the surface of the order $n = 4$ being arbitrary. On the contrary, when there is no surface of the order $n - 4$ which passes through the double and cuspidal curves of the given surface of the order n, this surface will belong to the genus $p = 0$. To this genus belong all the surfaces which can be transformed into a plane; which is the case for most of the surfaces that have been considered hitherto, and of which M. Chasles, M. Cremona and I myself have offered a fairly large number of examples.

»When the reciprocal surface of a given one belongs to the same genus, one has the special theorem:

«Given a surface of the order n and of the class k, the number of indeterminate coefficients of a surface of the order $n - 4$ passing through the double or cuspidal curves of the given surface is equal to the number of indeterminate coefficients of a surface of the class $k - 4$ which has for its tangent planes all the double tangent planes of the given surface.»