III. On the isochronous curve, on which a heavy body descends without acceleration, and on the controversy with the Abbé De Conti

When, in these Acta published in March 1686, I had brought out a demonstration against the *Cartesians*, in which the true estimation of forces is set forth and it is shown that it is not the quantity of motion, but potentia, differing from the quantity of motion, that is conserved, a certain learned man in France, the Abbé *De Conti*, replied on behalf of the Cartesians, but — as afterwards appeared — without having sufficiently grasped the force of my argument. For he believed that certain other received principles were being attacked by me, which he lists in the Nouvelles de la République des Lettres for June 1687, p. 579, and he denies (p. 519 ff.) that he recognises the contradiction which I seem to myself to find in them: although it never came into my mind to doubt those principles, as I reminded him in the Nouvelles de la République des Lettres for September 1687. In order to evade my objection, he had betaken himself to a side-issue of time, which — in the way in which the state of the controversy had been conceived by me — is plainly accidental. For, the height remaining the same, the same force is acquired or expended by heavy bodies whatever the time allowed, which is increased or diminished according as the inclination of the descent is greater or less. On that occasion, so that it might appear more clearly that time — and hence the distinction between isochronous and non-isochronous powers — is here nothing to the purpose, and that our dispute might yield some increase of knowledge, I proposed to him, in the said Nouvelles for September 1687, the following problem, solved by me in the course of writing and, as it seems, not inelegant: "To find an isochronous curve on which a heavy body descends uniformly — that is, in equal times draws equally near to the horizon — and so is carried downward without acceleration and always with equal velocity." But the Abbé *De Conti* made no further reply, whether because he was unwilling to touch the problem, or because, having at last understood my meaning, he judged himself satisfied. But in his stead the most celebrated *Christiaan Huygens* judged this problem worthy of his effort; his solution, wholly consonant with mine, appears in the Nouvelles de la République des Lettres for October 1687, though with the demonstration suppressed and the explanation of the distinction between the various lines of the same kind, as he says, which he notes as satisfying it. These things, therefore, I wished to supply in this place, and would have done so sooner, had I not been awaiting something here from the Abbé's industry.

Problem. To find a plane curve on which a heavy body descends without acceleration.

Solution. Let there be (fig. 116)[1] any semicubical paraboloid curve βNe (namely, one in which the solid under the square of the base NM and the

[1] *Editorial note:* The plate containing fig. 116 could not be located in the scans available for this work (the Gerhardt edition scan has no plates section); it is not reproduced here.

parameter aP is equal to the cube of the height βM), so placed that the tangent βM at the vertex β is perpendicular to the horizon. If at any point N whatever of this curve a heavy body be placed, endowed with that further speed of descent which it could have acquired by descending from the horizontal Aa — whose elevation $a\beta$ above the vertex β is $\frac{4}{9}$ of the parameter of the curve — then the same body will descend further uniformly along the curve Ne, however far continued, as was desired.

Demonstration. Let the straight line NT touch the curve βNe at N and meet βM at T. Then certainly (from the known property of the tangents of this curve) TM will be to NM in the subduplicate ratio of $a\beta$ to βM. Therefore TM to TN will be in the subduplicate ratio of $a\beta$ to $a\beta + \beta M$, that is, to aM. Now the ratio of TM to TN is the same as that of the velocity of descending along the curve (that is, of drawing nearer to the horizon along the curve) which the heavy body has when placed at N, to the velocity with which the same body would descend from N onward, not along the curve, but freely, if it could (as is clear from the nature of inclined motion). But this free velocity is further to a certain constant one in the subduplicate ratio of aM to $a\beta$; for (as is clear from the motion of heavy bodies) free velocities are in the subduplicate ratio of the heights (from which, by descending, they are sought), namely aM: therefore the velocity of descending along the curve Ne, which the body has when placed at any point N of the curve, is to the constant velocity in the ratio compounded of the subduplicate ratio of $a\beta$ to aM and of aM to $a\beta$ — which is a ratio of equality. That very velocity of descending along the curve is therefore constant, that is, everywhere the same on the curve Ne. Which was to be shown.

Corollaries: 1) A heavy body having a speed as though fallen from some height Aa can descend from the same point N along infinitely many isochronous curves, but of the same species — differing only in the magnitude of the parameter — such as Ne, $N(e)$, NE, which are all semicubical paraboloids, and therefore similar to one another. Indeed any paraboloid of this kind serves here, provided it be so placed that $a\beta$ (or $(a)(\beta)$), the distance of the vertex from the horizontal $a(a)$ from which the body began to descend, is $\frac{4}{9}$ of the parameter of the curve βe (or $(\beta)(e)$): nor does it matter whether the body, about to descend isochronously along the curve $N(e)$, has reached N from $a(a)$ by the path $(a)(\beta)N$, or by some other, or has acquired the same speed and direction without any descent at all, from some other cause. Yet of those infinitely many isochronous curves, along which the body can descend from N onward without acceleration, that one affords it the swiftest descent whose vertex is the point N itself — such as NE, which the straight line AN, perpendicular to the horizon, touches.

2) The time of descent along the straight line $a\beta$ is to the time of descent along the curve βN as half the height βM is to $a\beta$ itself: and accordingly, if βM be double $a\beta$, the times of the descents along $a\beta$ and along βN will be equal. The reason for this is manifest: for the times of uniform descent are to one another as the heights; and, from what has been demonstrated by *Galileo*, the time in which a moving body traverses the height $a\beta$ by accelerated motion is double that in which it traverses an equal height βM (as happens here, though

along the curve βN) by uniform motion having a speed equal to the last one acquired by acceleration at β.

I confess, however, that I proposed this problem not to the foremost geometers, who are versed in a certain deeper Analysis, but rather to those who side with *that learned Gaul*, whom my complaint about most present-day Cartesians (paraphrasts of the Master, rather than his rivals) seemed to have somewhat offended. For such men attribute too much both to the dogmas received among the Cartesians and to the analysis widely current among them, so much so that they suppose themselves able, by its aid, to accomplish anything whatever in Mathematics (provided only they are willing to take on the labour of calculation), not without detriment to the sciences, which are now cultivated too negligently out of a false confidence in what has been discovered. To these I had wished to offer, in this problem, matter for exercising their Analysis — a problem that needs not lengthy calculation, but art.

If, however, anyone should complain that the solution has already been snatched from him, he may seek another isochrone neighbouring on this one, in which the heavy body recedes uniformly not, as hitherto, from the horizontal (or approaches it), but from a certain point. Whence the problem will be this: *to find a curve on which a descending heavy body recedes uniformly from a given point, or approaches it.*

Such would be the curve NQR, if it were of such a nature that, from a given or fixed point A, straight lines being drawn to the curve at will, such as AN, AQ, AR, the excess of AR over AQ were to the excess of AQ over AN as the time in which the arc QR is descended is to the time in which the arc NQ is descended.