Derivation of elementary divisor theory from group theory.

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As is well known, elementary divisor theory gives, for modules of integral linear forms, a normal basis of the form $(e_1 y_1, e_2 y_2, \ldots, e_r y_r)$, where each e is divisible by the one following; the e are thereby uniquely determined up to sign. Since every Abelian group with finitely many generators is isomorphic to the residue class system modulo such a module, the decomposition theorem for these groups as a direct sum of largest cyclic [groups] is thereby proved as well. Conversely, the decomposition theorem is now obtained directly and purely group-theoretically, in generalization of the proof customary for finite groups, and from it elementary divisor theory is derived by passing from the residue class system to the module itself. The group theorem thus proves to be the simpler theorem; in the applications of the group theorem — e. g. Betti and torsion numbers in topology — going back to elementary divisor theory is accordingly not required.